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Procedia - Social and Behavioral Sciences 162 (2014) 477 – 486

Procedia
Social and Behavioral Sciences

XVIII Congreso Panamericano de Ingeniería de Tránsito, Transporte y Logística (PANAM 2014)

A Method Using GIS Integrated Voronoi Diagrams for Commuter Rail Station Identification: A Case Study from Brasilia (Brazil)

Diego Rosa Mota[a,*], Marise Takano[b], Pastor Willy Gonzales Taco[b]

[a]*Ministry of National Integration, SGAN 906, Asa Norte, Brasília, 70790-060, Brazil*
[a,b] *Postgraduate Programme of Transportation-PPGT, University of Brasilia, Asa Norte, 70910-900, Brazil*
[a,b] *Research Group: Comportamento em Transportes e Novas Tecnologias - CTNT*

Abstract

This article describes a station location method for a commuter rail system applying a GIS integrated Voronoi diagrams. The method comes from a previously defined track line and considers the stations area coverage. As parameters to define area coverage we first used the point density representing the maximum concentrated activity area. We also use the trip generating rate weights for point density. The method was applied to Brasilia Metropolitan Area, and the final product was a "T" Trunk-Feeder framework that allows an integrated transportation system planning considering others existing transit systems. The stations were classified according to their degree of activities density and its importance for integration. The "T" Trunk-Feeder is a representation of the commuter rail network and the Trunk-Distribution is the transport network responsible for distributing the passengers in central business district - CDB. The proposed model features 69 km extension, with the estimated total travel time of 62 minutes, and 17 stations being integrated into 2 metro, 4 integrated into LRT stations and 3 integrated into the bus stops.

Peer-review under responsibility of PANAM 2014.

Keywords: Rail Station location, Commuter Rail, Voronoi Diagram, Trip generator, Transport Integration, Brasília Metropolitan Area

1. Introduction

The Brazilian metropolises mobility problems are mainly set by a large flow of passengers who travel large distances because of the dependence on a center where most opportunities and jobs are located. Many metropolitan areas have experienced similar situations and found in the commuter rail a way to ease this problem.

In Brasilia, capital of Brazil, the growing demand, the increase of motorization rate and overcoming great distances are also a characteristic reality, especially in the Brasilia Metropolitan Area (BMA) south corridor. This area is a conglomeration of satellite cities over 60 km, which has 738.373 inhabitants and is located between two

* Corresponding author. Tel.: +55-61-20345471 ; fax: + 55-61-31071115
E-*mail address*: drmota.enc@gmail.com

Peer-review under responsibility of PANAM 2014.
doi:10.1016/j.sbspro.2014.12.229

distinct Brazilian States: Federal District and State of Goiás. In addition, there is a considerable level of social inequality since the population is marked by economic dependence of the central business district - CDB, where job vacancies are concentrated. According to the Urban Transportation Planning of Brasilia (PDTU, 2011), actually, more than 110.000 daily trips are made between these cities, but it is estimated there will be a 43% increase in travel by public transportation (bus) and 59% for individual vehicles trips during the morning rush hours until 2020. Therefore, the commuter railway line implementation discussions are under feasibility studies by the Federal Government. The proposal is to sharing parts of freight railway infrastructures with a commuter rail system.

For the deployment of a railway system Vuchic (2005) enhanced two essential factors: the track layout and the location of stations. This article stemmed from previous studies about track layout from University of Brasilia Post Graduation Program of Transportation and focused on station locations methods. The Voronoi diagrams algorithms are the most tradition method used to solve this kind of problems and actually many geographic information systems – GIS tools has applied them to identify locations.

Thus, the main aim of this paper is to propose a method using GIS integrated Voronoi diagrams for commuter rail station identification. With the strategic areas results, it was possible do define a framework for integration with the existing transportation network. In this way, we planned a model of efficient, integrated and sustainable transport that stand up to the mobility problems.

2. Review Literature

2.1. Rail Transit Station Locations

Stations represent locations at which passengers have access to the transit network. These elements often have a strong impact on their surroundings. Moreover, some operational issues like speed and travel time, riding comfort, operating costs are linked with the number and location of stations. All of these items indicate the locations of stations are key elements of a rail transit planning.

Vuchic (2005) suggests that a transit designer must be familiar with every aspect that is affect by the number and the location of stations. The planning is complex and must include quantitative and qualitative factor. Among the major objectives in planning station location we enhanced the coverage area, the promotion of integration and the political, economic and operational aspects.

The amount and location of stations are fundamental variables in an urban train operation system, since they are linked to the speed and time of travel, passenger comfort, operations costs, among other factors. From this fact arises the necessity of the existence of a plan to lease stations that qualitatively and quantitatively optimize various aspects suggested by Vuchic (2005). There are many techniques findings in the literature review considering that aspects. The Table 1 resumes the principal findings

Table 1. Methods for station locations

Author (s)	Definition
Vuchic (2005)	Area coverage is a direct result of density of stations. Maximizing area coverage is an important goal in rail network design because system usage depends on the ability of potential passengers to easily get to stations. As such, this makes area coverage by a station a circular surface with a radius of maximum walking distance (400m).
Horner & Grubesic (2001)	The authors propose an area coverage by a station a parabolic surface manly for *park-and-ride* station because it reflects the decision of the passengers, who choose to embark on a further station in the direction of your journey, rather than driving in the opposite direction to embark on a nearest station.
Novaes (2007)	Applied Voronoi diagrams to define the influence area of stations and to allocate them along a metro rail line.
Silva (2008)	Travel behavior theory based, the author developed a method for identify catchment area of a metro station applying an GIS integrated space-time prism model. Catchment area depends on the passenger decision of travel within a certain time.
Repolho et al (2012)	Propose a mixed-integer optimization model saving travel costs and determining numbers and location of stations for a high-speed train between Lisbon and Porto. Others variables were considered in this model, generating travel areas, OD matrix, catchment area and travel time.

Even though the methods in Table 1 consider at least a part of elements suggested by Vuchic (2005) for rail transit planning, they can be used together in techniques and spatial analysis software such as ArcGIS. For that, this paper focus on a GIS integrated method which allows special attention to major activity areas and trip generators along the rail track study.

2.2. Voronoi Diagrams

The Voronoi diagram is a useful tool to study geometric proximity in a plane. It allows the identification of coverage areas and regions of influence in space, and therefore applied to problems such as facility location and zoning. A Voronoi diagram is also known as a Dirichlet tessellation. The cells are called Dirichlet regions, Thiessen polygons or Voronoi polygons.

The primary concept of Voronoi Diagrams we found in Dong (2008): "The Given a set of a finite number of distinct points in the 2-D Euclidean space, a Voronoi diagram of the point set is a collection of regions that divide up the plane, and all locations in one region (exception the region boundary) are closer to the corresponding point than to any other point (Fig. 1).

The mathematically concept of the Voronoi diagram was found in Novaes (2007): "given a set of distinct points P ≡ {P1, P2, ..., Pm} in a continuous space, it seeks to involve all the other points of this space with the closed set of m points closest member of the set P is the set of Voronoi diagram generator, with $m \geq 2$. "

In this way, be μ (X, Y) the variable that expresses the distance between two points on a proper metric, and being $X, Y \in R^2$ points of the plan, the Voronoi diagram is defined mathematically by equation 1

$$V (P_1) = \{X \in R^2 \mid \mu (X, P_i) \leq \mu (X, P_j), j = 1, ..., m\}. \quad (1)$$

The simplest example of this formulation is the ordinary Voronoi diagram (Voronoi-OD), also known as Thiessen polygons, where the distance between two points is the Euclidean, with the resulting sub-regions formed by convex polygons. After this start up, other features of the diagram, called Voronoi diagram with multiplicative weights (Voronoi-MW), with additive weights, combined weight and power voronoi were developed as shows the resume in Figure 1.

More discussions on the concept of Voronoi diagram from both historical and geometric viewpoints can be found in Okabe et al. (1992, 2000).

2.3. GIS-Integrated Voronoi Diagrams

Voronoi diagram is an interdisciplinary concept that has been applied to many fields. In geographic information systems (GIS), existing capabilities for generating Voronoi diagrams normally focus on ordinary (not weighted) point (not linear or area) features.

Although theoretical and computational aspects of Voronoi diagrams have been extensively documented, there's still efforts have been made to bring Voronoi diagrams into GIS for practical use. There are 2 major issues concerning the integration of Voronoi diagrams and GIS: (1) many approaches to generating Voronoi diagrams have worked with point data outside GIS, and it is not straightforward to bring the results into GIS.

Then, for better integration of Voronoi diagram models and GIS, Dong (2008) developed a raster-based approach, and implemented seamlessly as an ArcGIS extension using ArcObjects known as Weighted Voronoi Diagram Extension for ArcGIS. The ArcScript was developed using Microsoft Visual Basic 6.0 and the ESRI 9.2 ArcObject.

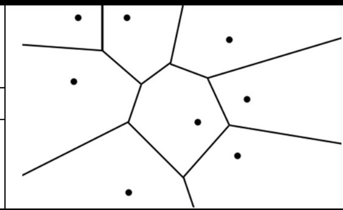
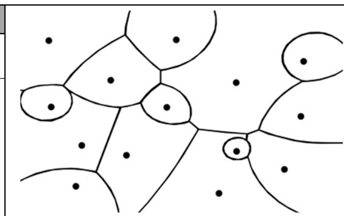
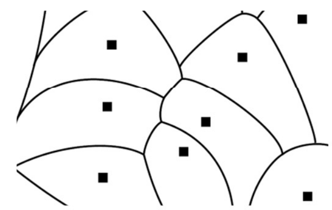
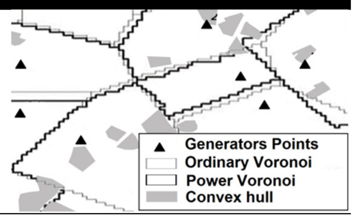
In this article, we chose this extension for three reasons: (1) The user friendly interface of ArcGIS; (2) the extension provides solutions for ordinary or weighted point, line or polygon features; (3) it can assign non-spatial attributes of input features to Voronoi cells through spatial joining

3. Methodology

Based on the Vuchic (2005) station location aspects, the method we developed considers the coverage area and integration aspects. The consideration of these two factors allows elaborating a planning of station location that can be detailed with the economic and operational aspects and OD matrix (see, for example, Repolho, 2012).

The method developed for an initial location plan resembles Novaes (2007), however we apply an integrated application into a geographic information system (GIS) with special attention to the possibilities of integration to the existing network. We also recognize the maximum coverage area through the identification of activities concentration points along the corridor.

For this purpose, the Method stemmed from a pre-defined track. Along this route are identified the Trip Generators and from them are produced the Voronoi diagrams that are weighted by the volume of trips generated by each one. The diagrams allow the concentration activities area identification along the rail track and then we identify the transportation network integration possibilities. Stemmed from it, we can propose a rail corridor framework. The method was divided into 5 steps: Database survey (Step1), Trip Generators identification and quantification (Step 2), GIS integrated Voronoi diagrams generation (Step 3), Network Integration analysis (Step 4), Framework proposed (Step 5).

	Ordinary Voronoi	
Applications	**Problems where applications Voronoi consider only the Euclidean distance**	
Function- μ(***X***, P_i)	$\lVert X - P_i \rVert$	
Boundaries	**Limits of straight line segments**	
	Weighted Voronoi	
Applications	**Problems that involve attributes reflected not only by the distance**	
Combined Weights		
Function μ(***X***, P_i)	$\frac{1}{w_{i1}} \lVert X - P_i \rVert - w_{i2}$	
Boundaries	**Fourth degree polynomial function**	
Multiplicatively Weighted Voronoi-MW		
Function μ(***X***, P_i)	$\frac{1}{w_i} \lVert X - P_i \rVert \quad , \quad w_i > 0$	
Boundaries	**Straight lines segments/ Apollonian circles**	
Addictively Weights		
Function μ(***X***, P_i)	$\lVert X - P_i \rVert - w_i$	
Boundaries	**Hyperbolic curve with focus P_i e P_j**	
	Power Voronoi	
Application	**Problems with barriers**	
Function μ(***X***, P_i)	$\lVert X - P_i \rVert^2 - w_i$	
Boundaries	**Straight lines segments**	

Source: Adapted from Novaes (2007); Devulapalli (2012); Youngxi et al (2012) and Carnasciali et al (2011).

Fig. 1 Voronoi Diagrams Functions

Step 1, the database surveying, is the influence area demarcation and the survey and mapping of spatial data through a Geographical information system-SIG, for example, ArcGIS software. At this stage the existing transport network such as bus lines, stops and terminals are identified and mapped.

Step 2, consists in a GIS trip generator mapping. Trip generators were used, hence National Department of Transportation - DENATRAN (2001) define it as, “places that significantly change the conditions for the movement of people and vehicles in the road system of the surround areas, as well as the pattern of trips in its region of influence”. Then, for the method, trip generators information such as schools, health centers, universities, malls and supermarkets in each of the cities through the rail track were identified.

In addition to the diversity of trip generators along the line it is necessary to identify its quantitative behavior in the travel generation. Thus, we use DENATRAN (2001) equations that allow the travel volume (V) measurement at peak hours through determined variable, as Table 2.

Table 2: Trip Generating Rates

Trip generator	Source	Variable Used	Equation
Schools	National Institute of educational research	Num. classrooms (NSU)	V = 22.066 NSU + 102.186
Universities	Secretariat of colleges	Num. students (NA)	V = 0.432 NA – 106.303
Hospitals	Brasilian Health system database	Num employees (NF)	V = 0.483 NF + 36.269
Supermarkets	Survey *on the spot*	Total Built-up Area (AC)	V = 16.53/100 m^2 of AC
Malls	Offices of the Malls	Gross Leasable Area (ABL)	V = 1732.7276 + 0.3054 ABL

Following the method steps, the GIS integrated Voronoi diagrams are generated. This stage allows the visualization of concentration activities areas that are possible sites to be train stations. To achieve this, we used the Multiplicative weights Voronoi diagram (Voronoi-MW), which allows the consideration of a weight that shows the equivalence between the attraction of each point generator and the demand attraction for the transport system studied. The weighting was done by each trip generator travel volume (V). It is important to know that the choice for the Voronoi-MW was done not only for the possibility of a weight consideration but also because ArcGIS is a software with a user- friendly interface, through the *Arc Script* developed by Dong (2008).

The method is pursued with the Step 4, examination of the possibilities of integration, where is analyzed the existing transport network mapping-lines and bus terminals, LRT and metro stations, among others – within the areas identified in the previous step.

Finally, in the Step 5 the framework train corridor is proposed, which depending on the characteristics raised in the previous steps, may take trunk shape, circular shape, among others.

The methodology was applied for the Brasília Metropolitan Area south corridor. We applied it for two situations. The first one, the rail track line existing e, secondly, for a rail track line proposed, shown in Figure 6 as well as the cities along the railway.

4. Results

The Step 1 method application provides the following data: Bus stops and terminals points mapped and buses routes provided by the Transportation Manager Department of Brasilia and the transportation network provided by the Secretary of State for housing, urban development and Regularization of the DF. We also mapped in ArcGIS 10.1 the railroad track and the influenced urban area.

Step 2 consisted in identifying the PGVs and knowledge of its quantitative behavior in the generation travel. For this, we use the equations of DENATRAN (2001), as Table 2. Assuming the location of trip generators and their value of travel volume (V), with the method step three, were generated the corresponding Voronoi diagrams with multiplicative weights for each trip generator category separately (Figure 2).

In order to identify areas of greater trip generator concentration and influence, which are potential locations for stations, the procedures of Voronoi-MW generation were repeated for all trip generators together and for all trip generator without shopping malls, as they showed high values for the volume of trips to other trip generator, resulting respectively in Figure 3(a) and in Figure 3(b).

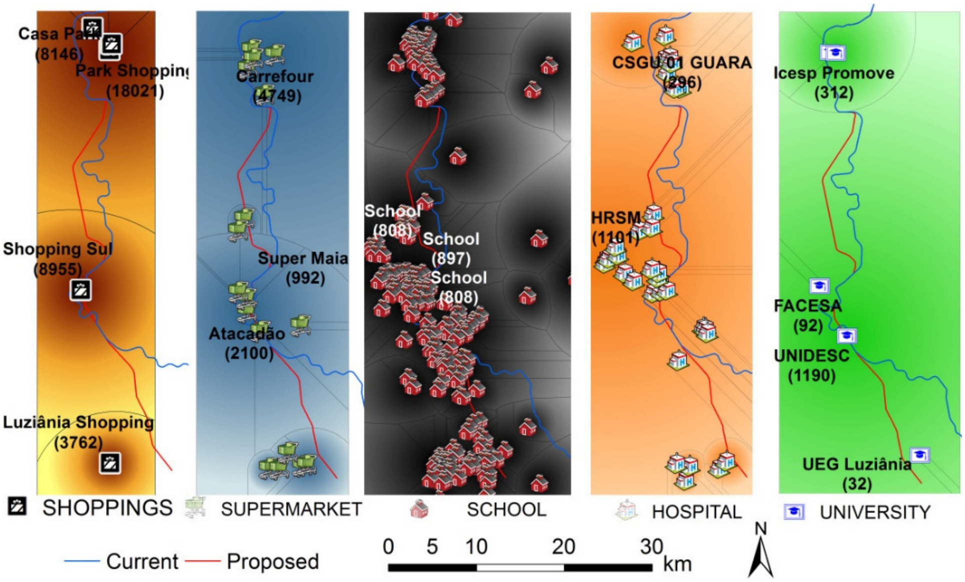


Fig.2 Voronoi diagrams-MW for each category of trip generator

The diagram of Figure 3(a) permits the visualization of three main regions of trip generator influence. The identification of secondary activities concentration areas, which are identified on the ellipses of 1 to 5, is also allowed by Figure 3(b).

After the activities concentration areas identification in the previous step, it was made the analysis of the integration points. For this, we identified the components of the transport network in the region, such as bus lines, the DF metro line, BRT and LRT line (provided in the Urban Transportation Planning of Brasilia) and the routes that cross the train track and can be used as feeder routes. This is illustrated in Figure 4.

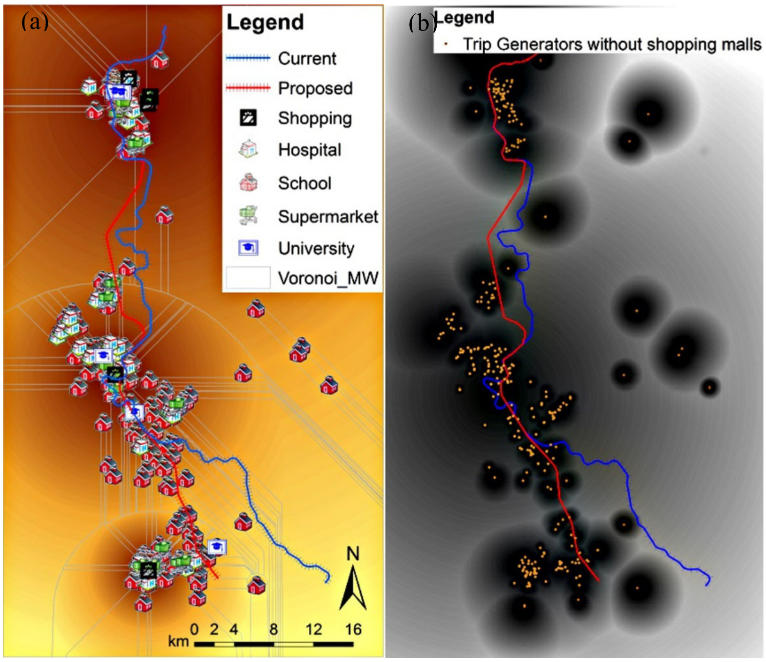


Fig 3 -Voronoi diagram MW

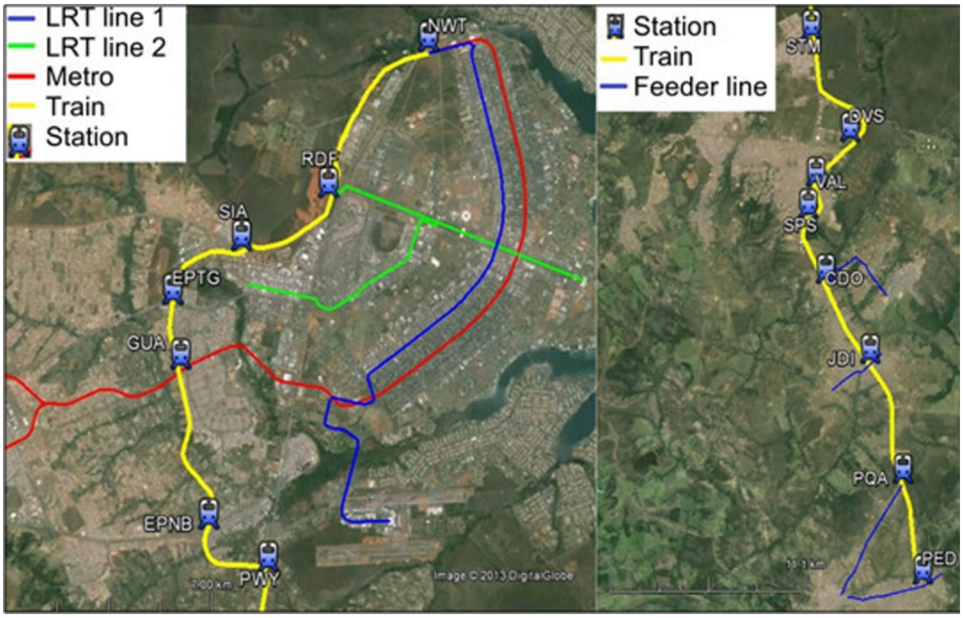


Fig. 4 Potential areas of integration

5. Discussion

As the results show in Figure 2, the first point to be noted is about the reliability of geometric shapes with the presented in literature about Voronoi diagrams. It is noted, for example, in the figure of the malls that the trip generator called "Luziânia Shopping" and "Shopping Sul" has values of V, respectively, 3762 and 8995. The boundary´s area of influence is a curve that is part of a circle (the circle of Apollonius) and remain concave for Luziânia Mall, which has smaller value of V and convex to the South Mall, with greatest weighting value.

In addition, it is known that when many points are analyzed together, the diagram is built in an iterative method, where the area of influence of each generator will be delimited according to the relative behavior of each of the points with the points around, which is reflected by the ratio k between the weights.

Thus, there are two categories of diagrams that can be analyzed: the first type, has the value of k (relationship between weights) near one, which corresponds to the diagram feature represented by insignificant variation between values of the weights. In this category, the points diagram generators have a well-defined region of influence, which reflects the local influence of these points.

The best example of this category it is the schools diagrams (Figure 2). Because of the large number of points and also due to the low difference between the weighting of the points values, the area of influence of each school is well-defined.

The second pattern is configured by the large difference between the values of weight, resulting values of k almost zero or very high. In this case, it is possible to identify the larger generator point weight exercising great influence under the remaining points, overshadowing them and distorting radially diagram lines.

This phenomenon is observed in health diagram (Figure 2). Due to its high value of V (1101 generation trips), the hospital named HRSM distorts the boundary lines nearest Hospitals, which has its region of influence decreased. It is possible observe the same aspect at points close to Carrefour and the Park Shopping and universities diagrams.

Another aspect noted by the discrepancy between weights it is the fact that the point of greatest weight encompass smaller weights points closer. This happens because there is a high difference among the values of weighting that the point of lowest weight not have any relevance in the region. This can be noticed by the points included by HRSM and at some points of the diagram of the schools.

In turn, in the Voronoi-MW diagram for all trip generators - Figure 3(a) - it is possible to notice three predominant region influences, delimited primarily by malls, which have, the largest travel volume among all trip generator (V between 4.000 and 18.000 trips).

The first region of influence is represented by the larger circle and is mainly bounded by the presence of the Park Mall (V = 18020), the Mall House Park (V = 8146), Florida Shopping (V = 2641) and Carrefour Hypermarket (V = 4748). They have the highest absolute values of the region, exercising influence across the study area, as can be noticed by the great circle and the distorted lines. The second region of influence, located in the city of Valparaiso are delimited by Shopping Sul (V = 8954) and Hypermarket Atacadão (V = 2100). The third circle is located in the central region of Luziânia and Luziânia Shopping (V = 3761) as the main trip generator. This region is not intercepted by the current train line, which indicates a need for review of the track line.

These three large regions of influence make it clear that in the region covered by the train, three large areas should be considered. These regions are candidates for the main stations of the future urban commuter train line.

The high malls V value facing the low V value for other trip generator makes analyzing the Voronoi diagram-PM without the malls a necessity- Figure 3(b). In this diagram, it is possible to identify well-defined regions of influence, allowing the identification of other possible areas of location of station outside those determined by malls, which are shown on the ellipses 1 to 5.

The location of the stations in the areas identified by the Figure 3(a) and (b) must be done by analyzing aspects and possibilities of integration offered by terminals, metro, LRT and bus lines in the region and raised in methodological Step 4. The principal points stations are: NWT, integration with the metro expansion line, LRT and BRT; RDF and SIA, buses and LRT link; GUA, metro integration; PWY, LRT and Brasília International Airport link. The other stations intercept roads and avenues that may be used by feeder and distribution bus lines.

5.1. Framework proposal

The framework proposed consists of a trunk-feeder structure shaped as a "T", (Figure 5), where the stations are classified according to their degree of activities densification and its importance for integration. The main axis of the "T" is a representation of the urban train line and the axis of distribution represents the subway lines, buses and subways, responsible for the distribution of passengers in the Brasilia CBD.

For the framework are proposed 4 levels of stations. The first three levels correspond to the three great circles generated in the Voronoi diagram-PM shown in Figure 3(a). The fourth level corresponds to those stations located outside the main region of trip generators influence, where the points are rare and sparse. This stations has as main characteristic the access by car (*park and ride*) or feeder line bus.

Within each level, there are still, main and secondary stations. The main stations are those located in areas of high concentration activities, and are characterized by a predominantly access on foot. Secondary stations may have access on foot and by feeder line bus. The all stations framework is shown in Figure 6.

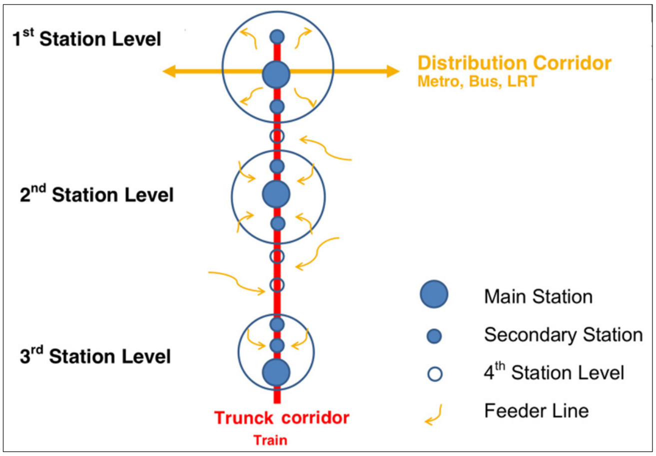


Fig 5 –Conceptual framework for an integrated transit system

The **1st level stations** are important in the framework because they are represented by the last line stations where passengers will arrive (considering commuting arrangements at morning rush hour) in order to proceed to the final destination, mainly in the Brasília CDB. At this level there is a main station represented by the metro integration station (GUA) and secondary others that still preserve their passenger distribution function by bus or LRT lines.

The **2nd level stations** are located in the city of Valparaiso, coming from the central circle of Figure 3(a). The main station is SPS, near to the Shopping mall. The **3rd level stations** are located at the beginning of the line, in the city of Luziânia. The main station (LZA) is located in the central region of the city, however further study of demand should confirm the need of a station in the Centre of Luziânia. The **4th level stations** are the stations PWY, JDI, STM, PQA and PED.

Therefore, considering the proposed route (with the Extensions to the Northwest Sector and to the Centre of Luziânia), we have the final Framework shown in Figure 6 and Table 3, that brings the operational estimating of the line with the values for the proposed extended stroke and for the current stroke to the Western City Station.

The proposed track line has a total of 17 stations, length of 75 km and estimated travel time of 62 minutes. The current track line has a smaller coverage area, with 11 stations, length of approximately 62 kilometers and estimated time travel of 53 minutes. These values indicate that the proposed track is better suited to the current, which presents greatest extent (due to its sinuosity) and a smaller coverage area.

Time values are presented considering information of average speed of urban train (V = 70 km/h, whereas the charts) presented by Fleming (n.d). It is suggested that a detailed study be made of the amount of stations and spacing, as indicates Vuchic (2005), in order to determine the speeds and travel times to the line. This study should consider the still the O.D matrix in the region.

Table 3. Station Specification

Specifications	Proposed Track	Current track
Total length of the stroke (km)	75	61.67
Total Stations	17	11
Average Spacing (km)	4.68	6.16
Density of stations (sta/km)	0.23	0.18
Total estimated time (min)	62	53

6. Conclusion

The method is suitable for localization studies of urban commuter train stations, because it facilitates the identification of areas of concentration and attractiveness of a region's activities, in addition to the possibilities of integration offered by transport network. The method also shows an important step in the planning process of urban rail lines that still must consider the economic and operational aspects of your deployment and the regional OD matrix.

The application of the method for the southern circuit of the BMA results in a transport framework that can cope with the growing demand for transport on the southern circuit of BMA and can directly benefit approximately 730,000 inhabitants. The template still shows an alternative integrated transport which can improve access by residents of the cities in the State of Goiás to the Brasilia CBD, besides being a sustainable proposal that diversifies the options for individual transport users.

As a proposal for future work, it is suggested the update of the OD matrix where travel demand is considered separately for each of the cities of the region. It is also suggested the development of an optimization model taking into account this demand along the track with economic and operational aspects of the urban train line to be built.

Acknowledgements

The authors are grateful to the Research Group of Travel Behavior and New Technologies (CTNT) of the Postgraduate Programme of Transportation of the Department of Civil Engineering - University of Brasília

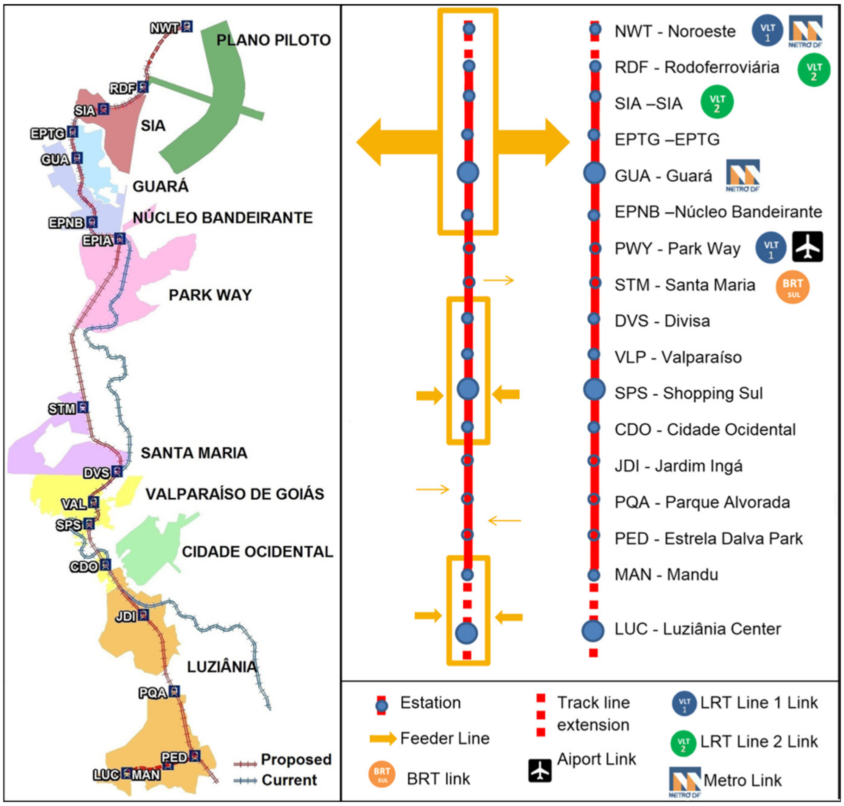


Fig 6 – The south corridor of BMA and the final integrated transit framework

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