THE DETERMINATION OF A MATHEMATICAL MODEL TO ESTABLISH THE BEST FARE STRATEGY

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ABSTRACT

This article develops a method to establish the best fare system to be implemented in a particular metropolitan region. The method was developed by collecting, analyzing and comparing data on the fare systems of cities in North America, South America, Europe, Asia and Oceania through a mathematical model. From each selected location the following data were gathered: population, area, population density, transportation systems, fare structure, pricing system, average income, average fare and relative fare index (RFI, an indicator formulated to reflect the relative ticket cost considering the local income level). Analysis of the data and graphs to test each criterion showed a relationship between the RFI on the one hand and the railway extension and population density on the other. The mathematical model entailed linear regression, using variables such as population density and railway extension. The regression result obtained is equivalent to the estimated RFI. Depending on the RFI value obtained by the mathematical model and the location of the metropolitan region in question (in a developed or developing country), planners can determine whether changes are needed to improve the fare system.

Keywords: Transport Planning, Pricing Strategies, Fare Structure.

INTRODUCTION

The market and social needs associated with collective transport are undergoing great transformation. On the market side, competition has increased with the presence of informal and alternative transport. Private car use has also increased, attracting riders formerly served by public transit systems. Even with the predominance of users from the middle and lower income segments, the tendencies on the demand side are for greater segmentation. From a standpoint of social needs, the contingent of poor people in developing countries is increasing due to the influx of rural migrants. These people often cannot afford the fares charged and are forced to commute on foot (Associação Nacional das Empresas de Transportes Urbanos, 2005).

Against this backdrop, the role of fare policies is fundamental to structure and improve urban mobility. Transit fares are a major factor in attracting passengers. They are also a basic element of transit system operations, affecting the financial condition of the transit agency. The fare amount, its relationship to the service quality and the convenience of fare payment greatly influence ridership. Types of fares and their collection also affect the efficiency of operations. The revenue collected from fares influences the method of financing transit operations in an urban area. Finally, in the long run, fares often have a significant impact on the form and development of central cities, their surrounding areas and suburbs. Therefore, planning fares for a given transit system requires careful consideration of numerous interrelated aspects of fares (Vuchic, 2005).

This study presents a methodology that aims to establish the best fare system to implement in a given location, through a mathematical model.

LITERATURE REVIEW

According to TRCP/TRB[1], a transit agency's fare policy establishes the principles and objectives that guide the fare decisions. This policy can be affirmed through a declaration. If a formal declaration is made regarding the fare policy, it should present long-term goals and identify more specific short-term objectives, as well as specify the orientations or procedures to determine and implement changes in the fare structure or system.

Decision-making scenarios

A range of approaches are used to make specific planning decisions. Some agencies use a top-down approach, starting with the establishment (or reconsideration) of the policy objectives and then identify and assess potential technological and structural options referring to these objectives. Other agencies decide first on changes in technologies or equipment and then consider the fare structure that can be established to use the new equipment. In other cases, an entity reaches decisions on strategy, structure and technology, guided by a change in the system (e.g., introduction of a new mode of service or significant expansion of existing service).

A recent analysis of fare planning indicates that agencies' decisions reflect three factors:

1. Policy: The agency has established a set of goals and objectives and seeks a new fare structure, new fare technology, or both to address specific goals. These goals can be short term, such as surviving an immediate budgetary crisis, or long term, such as improving public mobility. The goals and the resulting strategies are usually

[1] TRCP Report 10 – Cooperative Research Program / TRB – Transportation Research Board.

agency-specific, but a growing number of regions are developing new technological and revenue-sharing approaches to facilitate regional coordination.

2. Technology: The agency has selected a new technology and develops a new fare structure to take advantage of the capabilities of this technology.

3. Service: The agency is introducing a new mode of service (e.g., light rail) and needs new technology, a new fare structure, or both for the new mode, and possibly for the overall system.

The decisions reached and the questions involved can differ considerably in function of these factors. For example, if policy is the decisive factor, then the process is interactive and requires analysis of tradeoffs and interrelationships between technology and strategic options. Evaluation criteria that reflect current objectives should be established for both types of options. Evaluation criteria that reflect the current objectives must be established for both types of options. In counterpart, if technology is the decisive factor, the fare structure typically is not considered until the new technology is chosen and the capacities and limitations of the new equipment are understood. In this case, the policy goals should be established, or at lease reviewed, for the development of a new structure. Finally, if service is the decisive factor, the scenario is similar to that of the policy factor, where both technology and strategy are considered. However, this scenario also requires establishment or review of the policy goals of the new service.

These scenarios are general. The decision-making process differs considerably from agency to agency. The specific process is affected by the size and complexity of the system (e.g., number of different modes), the existence of a fare structure and system and institutional configuration (e.g., number and nature of entities and sources of financing and legal requirements), the governmental situation (including the size and type of policy, as well as the organization of the agency and its staff) and the nature of “external influences” (e.g., local interest groups, businesses and news reported in the media). The themes considered most important in reaching fare decisions also vary.

The process described in Figure 1 is an idealized decision-making process. Not all decisions will be reached by following every step of this process. The steps shown in Figure 1 generally follow a policy planning and service approach.

In a scenario based on technology, the agency should have completed the steps between “evaluate fare system options” and “select fare system”. The agency should also carry out the other steps presented, including “defined and prioritized fare policy goals” and “develop evaluation criteria”. It is clear in this case that the objectives and criteria will only be applied to questions regarding the fare strategies and structures.

Depending on the scenario and decision reached, the agency can proceed with only some of these steps, and not necessarily in the order suggested in Figure 1. However, this process includes all the steps a transit agency probably will carry out.

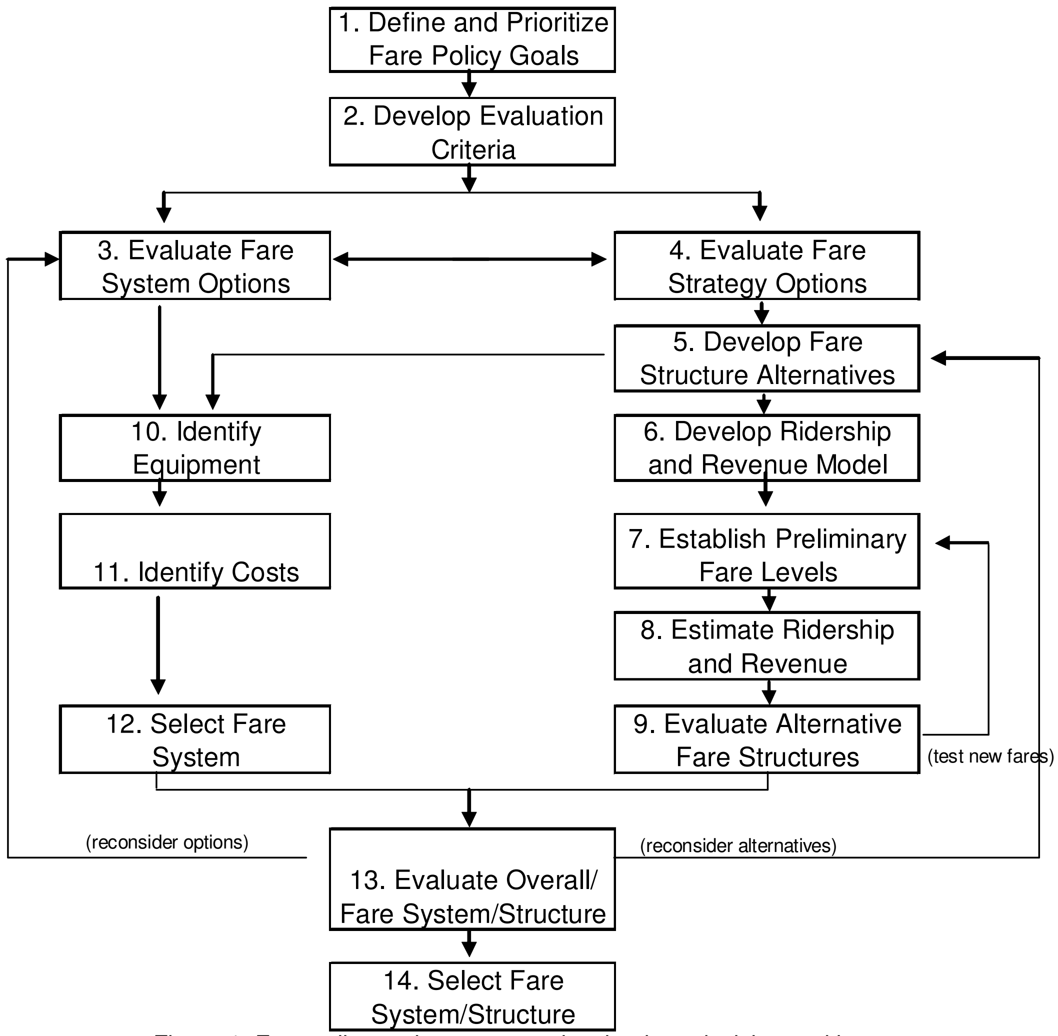

Figure 1: Fare policy and structure and technology decision-making process.

This study does not consist or development of the decision-making process presented in Figure 1. The reason is the need to determine the real objectives, targets and other definitions, which cannot always be assumed. It is up to the government, through its transport planning entities, to follow the flow chart presented in Figure 1 after carrying out a study like the one proposed here.

FARE POLICY

According to a study by the National Association of Urban Transport Companies of Brazil entitled “New Fare Policy Trends” (NTU, 2005), the fare structure is an important part of urban planning policies because it has direct effects on the socioeconomic condition of users, land use patters and the financial sustainability of transportation systems.

In formulating fare policies, three aspects must be considered (Figure 2):

1. Objectives: the results expected from applying the policy;

2. Fare structure: ways of charging for the services, involving the price level, fare collection strategies and payment options;

3. Payment technologies: tools (equipment, procedures and programs) used for sale of tickets and control of fare payment.

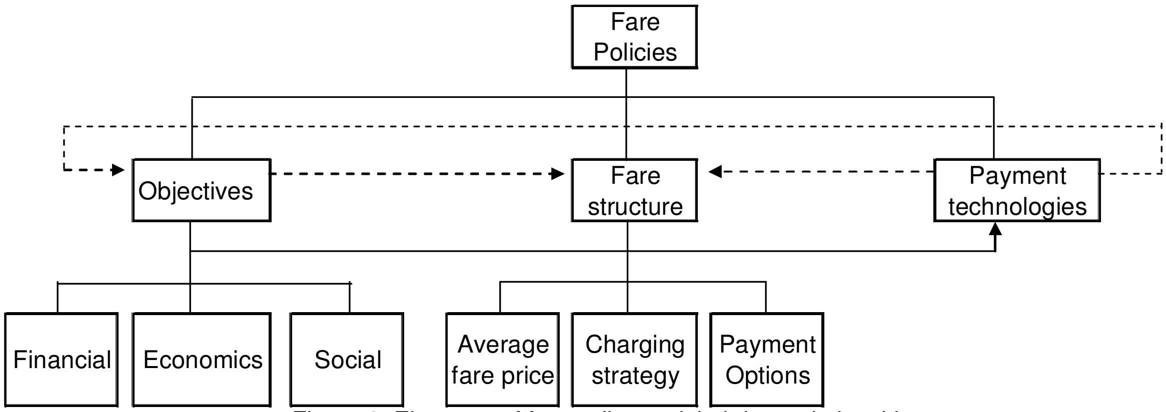

Figure 2: Elements of fare policy and their interrelationships

Objectives of Fare Policies

There are three basic objectives of fare policies:

1. Financial: to cover the cost of services;

2. Economic: to induce economically optimal user choices;

3. Social: to redistribute income and foster inclusion of less favored classes.

The existence of a mass transit system adequate to the characteristics of the population (in general and riders in particular) and the existing infrastructure is fundamental for the sustainable development of a local economy.

Fare Structure

According to the policy guidebook on fare structures from the Institute for Transport Studies, University of Leeds, available from the Knowledgebase on Sustainable Urban Land Use and Transport (KonSULT), fare structures are important policy instruments because of their potential impact on:

a) Efficiency: If a fare structure encourages transfers from cars, then it will affect traffic congestion and increase efficiency of labor markets due to increased access to jobs and possible reduction in unproductive travel time.

b) Livable streets: Reduced traffic levels make streets more livable.

c) Protection of the environment: Reduced levels of local traffic cut air and noise pollution, put less pressure on natural resources such as oil and green space and reduce greenhouse gas emissions.

d) Equity and social inclusion: Fare structures can impact the affordability of public transport and improve access to key goods and services by socially excluded and less well-off citizens.

e) Safety: Traveling by public transport is much safer than by car for passengers and also reduces the number of accidents suffered by pedestrians and cyclists.

f) Economic growth: If a fare structure encourages transfers from cars, then reduced traffic congestion can stimulate economic growth and improve access to jobs.

g) Finance: Fare structures can have a significant impact on revenues and also on costs because they can influence the level of capacity required.

The fare structure is composed of three elements, which together define the bases for charging for transportation services. They are:

- Average fare price: the method to determine fares and the procedures for their adjustments over time (in this work we do not consider this aspect).

- Charging strategy: falling basically into two categories – unified and diversified, in the latter case considering questions of integration, discounts and free passes.

- Payment options: conditions offered to users to pay fares (single ticket, prepaid electronic card, postpaid billing, etc.).

Charging strategies

The charging strategies are basically divided into two fare structure categories: unified and diversified.

A unified fare is a single price for any trip in a transportation network. A diversified fare structure means there are different prices depending on the type of user, quality of service, trip length and/or travel timing (peak/non-peak, etc.).

According to the American Transportation Research Board (TRB) and the National Association of Urban Transport Companies of Brazil, the different types of strategies can be summarized as follows:

- Flat fare: a single fare is charged for any trip within the transport network.

- Distance or zone: different fares are charged according to the distance traveled or number of zones covered.

- Market: the use of unlimited, weekly, monthly or annual passes, establishing a frequency of use.
- Time: the fare is different depending on the time of day (peak versus off-peak hours) or on weekends and holidays.
- Service: the fare is different depending on the type of transport utilized (such as bus or train) or according to the speed (normal versus express).

Table 1 shows the main advantages and disadvantages of each fare system, as pointed out by Pitcher (2003).

Table 1: Advantages and disadvantages of fare strategies

Fare Strategy Options		**Advantages**	**Disadvantages**
Flat Fare	**Flat Fare**	Esiest to understand	Places inquitable burden on those making short trips.
		Simpliest and least expensive to implement and administer	Increase will cause greatest loss of riders
		Lowest level of fare abuse	
Differentiated Fare	**Distance/ Zone-Based**	Should produce greatest revenue	Difficult to use
			Difficult to implement and administer;may require special equipment.
		Considered equitable; longer trip has higher cost.	Potentially high level of fare abuse.
			May be unpopular with users with long trips.
	Market-Based	Generally considered equitable;offers ability to pay less.	Generally produces least revenue.
		Can minimize ridership loss with fare increase.	Potentially high level of fare abuse
		Maximizes prepayment.	Requires extensive marketing to maximize ridership.
		Most convenient option.	Highest media production and distribution cost.
	Time-Based	Should increase ridership	Potential for conflicts with drivers
		Allows management of fleet usage through shift to off-peak.	Potential for fraud (agents on rail)
		Considered equitable; commuters pay more.	May require equipment modifications (or new equipment)
	Service-Based	Relatively easy to understand.	May be unpopular among users of higher cost service.
		Considered equitable; higher quality or higher priced service has higher cost	
		High revenue potential; low fare abuse	Complicates transfers (e.g, may require payment pf "upgrade" fare in transferring).
		Allows managment of fleet usage through shift between services.	

Payment options

A variety of payment options are available, the number of which has increased with advances in information technology. The most common options are:

1. Single ticket: This scheme entitles users to one trip or access to an integrated transport system. Generally the unit price is more expensive.

2. Multiple ticket: This scheme entitles users to several trips or accesses to an integrated system. The initial outlay is higher but the unit price is generally lower because of the number of rides acquired.

3. Time pass: This entails magnetic tickets or smart cards (with chips) allowing an unlimited number of trips within a defined period (month, week, day or number of hours). It can also consider complementary payment in case of transfer between transport modes (e.g., bus to subway) or trips between different areas of a greater metropolitan region.

4. Prepaid credit: In this case the smart card is loaded with a determined fare value and the fare is deducted from the balance each time it is used. The option is most suitable for system with differentiated fares.

5. Postpaid service: The use is monitored by a smart card and billed afterward through an account sent to the user's residence or office.

ANALYSIS OF THE FARE SYSTEM OF SOME METROPOLITAN REGIONS OF NORTH AMERICA, SOUTH AMERICA, ASIA AND OCEANIA

The data for this study were obtained from a review of the literature, research of socioeconomic data and information on the mass transit systems in various locations.

In choosing the locations included in this study, we considered criteria related to the cultural, political and economic importance of each one in its wider region as well as the experience of the second author in some of the cities selected.

For each location (metropolitan region) selected, we gathered the following data:

1. Population.

2. Area.

3. Demographic density.

4. Transportation system, including extension, number of lines, number of vehicles and number of passengers carried.

5. Fare integration (total, partial or none).

6. Predominant fare system.

7. Average per capita income in the region (according to the exchange rate of the region's currency with the U.S. dollar on December 29th of the year for which the information was obtained).

8. Weighted average monthly fare, determined by the weighted average between the number of passengers and the respective fare (passengers1 * fare1 + passengers2 * fare2 / passengers1 + passengers2). In this case we considered one month of use, so for cities without a fully integrated system we considered a single fare for each transport system, and then calculated the weighted average, multiplied by 44 (considering that a typical month has 22 work days and most users commute to and from work each day, making two trips). In these cases, we used the exchange rate with the dollar of July 1, 2008.

9. Relative fare index (RFI), an indicator we created to reflect the relative cost of public transportation in riders' budgets, calculated as the average monthly per capita income in the region divided by the weighted average monthly fare.

We chose metropolitan regions in developed and developing countries of North America, South America, Europe, Asia and Australia, namely: Federal District of Mexico (Mexico), New York-New Jersey Metropolitan Region (United States), Recife Metropolitan Region (Brazil), City of Santiago (Chile), São Paulo Metropolitan Region (Brazil), Brussels Capital Region (Belgium), Greater London (England), City of Madrid (Spain), City of Moscow (Russia), Ile-de-France or Greater Paris (France), Porto Metropolitan Area (Portugal), Melbourne Metropolitan Area (Australia), Seoul Metropolitan Area (South Korea) and the City of Tokyo (Japan).

Table 2 presents the characteristics of the metropolitan areas selected. We should mention the difficulty of standardizing the areas chosen for comparison and of harmonizing the political and urban divisions with the transport system.

Table 2: Characteristics of the metropolitan areas selected

Location	Area (km^2)	Pop. (mi) 2006	Demog. density (inhab/km^2)	Transp. system	Existing transport system: Ext. (km)	No. of lines	No. of vehicles	No. of passengers carried (year - million)	Fare integration?	Predominant fare system	Average per capita income (US$)	Basic Fare[#] (US$)	RFI observed
Mexico City (DF)	1479.0	8.8	5,965	Bus	3,519.7	89	1,166	290.9	None	Service	135.72	8.36[a]	16
				Subway	176.8	11	355	1,417.0					
				Trólebus	453.1	17	330						
				Tren Ligeiro	n.a.	1	16	62.9					
New York - New Jersey metropolitan area	10,101.0	8,0	1,700	Bus	8,998.4	553	n.a.	1,010.9	Partial	Flat Fare	4,050.00	76.97 [b]	53
				Subway	368,0	27	6,494	1,499.0					
				Train	2,688.9	n.a.	n.a.	221.7					
				VLT	67.8	n.a.	n.a.	16.4					
Recife metropolitan area	2,768.0	3.7	1,320	Bus	n.a.	354	2,700	436.7	Total	Zone	400.00	37.40	11
				Subway	29.3	3	n.a.	69.3					
				Train	31.5	1	n.a.	n.a.					
Santiago	876.8	4.6[p]	5,324	Bus	n.a.	n.a.	4,654[s]	n.a.	Total	Flat Fare	551.47	36.08[c]	15
				Subway	84.4	5	n.a.	330.0					
				Train	65	n.a.	n.a.	n.a.					
São Paulo Metropolitan area	8,051.0	19.7	2,450	Bus	n.a.	1,860	18,761	3,033.8	Partial	Flat Fare	711.44 [s]	68.64[d]	10
				Subway [s]	61.3	4	702 [t]	611.0					
				Train	257.5	6	110	584.0					
Brussels Capital region	241.5	1.1 [o]	4,555	Bus	420.4	47	598	76.1	Total	Flat Fare	1,115.30 [r]	65.51[e]	17
				Metrô	49.9	3	90	122.5					
				Tram	217.3	18	311	70.8					
				Train	72.5	n.a.	n.a.	n.a.					
Greater London	1,579.0	7.5	4,758	Bus	n.a.	700	8,000	1,168.0	Total	Zone	4,233.38	185.44[f]	23
				Subway	408.0	12	4,070	1,014.0					
				Tram	28.0	3	24	n.a.					
				DLR	31.0	n.a.	94	61.3					
				Train	788.0	n.a.	n.a.	232.0					
Madrid	604.3	3.2	5,304	Bus	3,485.0	204	2,022	490.6	Total	Zone	1,700.40 [r]	63.85 [g]	27
				Subway/VLT	220.0	13	n.a.	657.4					
				Train	101.0	n.a.	n.a.	n.a.					
Moscow	1081.0	10.44	9,660	Bus	15,044.1	1.247	10,499	1,882.9	Partial	Flat Fare	1,244.16[o]	55.42[h]	22
				Subway	278.8	13 [r]	n.a.	2,475.6					
				Tróllyebus	940.6	85	1,601	465.5					
				Tram	415.1	38	917	275.0					
				Train	782.1	n.a.	n.a.	605.6					
Ile-de-France[q]	12,012.0	11.49[q]	957	Train	1411.0	7	4,87	1,051.8	Total	Zone	2,885.20 [r]	86.98[i]	33
				Subway	211.3	16	3,553	1,335.7					
				Tramway	23.5	2	235	44.1					
				Bus	22,820.6	1.334	4,064	1,191.0					
Porto metropolitan area	814.5	1.27	1,571	Bus	496.0	94	508	190.0	Total	Zone	1,109.70 [r]	37.08 [j]	30
				Subway	58.9	5	n.a.	38.6					
				Train	35.5	n.a.	n.a.	n.a.					
Melbourne metropolitan area	8,097.2	3.59	443	Bus	n.a.	314	1,472	80.0	Total	Zone	1,853.7	99.51[k]	19
				Tram	249.0	27	499	155.0					
				Train[s]	382.0	15	329 [v]	162.0					
Seoul metropolitan area	605.0	10.29p	17,019	Bus	n.a.	n.a.	n.a.	1,699.0	Total	Distance	2,139.70 [r]	33.51[m]	64
				Subway	286.9	8	399	2,023.8					
				Train	246.0	n.a.	n.a.	704.5					
Tokyo city	621.05	8.57	13,720	Bus	779.0	138	1,467	206.0	Partial	Distance	4,856.8	78.76[n]	62
				Subway	289.4	12	n.a.	2,929.8					
				Train	310.6	n.a.	n.a.	2,681.9					

&Number of units of a specific ticket.
n.a. Not available.
Rate for the year 2008.
a44*weighted price.
b Rate for one month.
c 44 Tickets at peak time.
d 44*weighted price.
e Unlimited monthly pass.
f Unlimited monthly pass (Zona 1 and 2).
g Unlimited monthly pass (Zona 1 and 2).
h Unlimited monthly pass - subway.
i Unlimited monthly pass (Zona 1 and 2).
j Unlimited monthly pass (Zona 1 and 2).
k Unlimited monthly pass (Zona 1).
m Unlimited monthly pass (distance - 10 km).
n 44*weighted price (from 7-11Km).
o Data from 2000.
p Data from 2002.
q Data from 2004.
r Data from 2005.
s Data from 2007.
t Each train has 6 cars.
u With monorail.
v Trains has 3 cars.

ANALYSIS OF THE MAIN CHARACTERISTICS OF EACH METROPOLITAN REGION

Some observations can be made from analyzing the data on each region selected.

A comparison of land area and the rail transport system of the metropolitan regions chosen shows that 78% of those with area over 1,000 km^2 have a rail system that carries the most passengers (train, light rail and/or metro). The only places where a rail system does not carry more passengers than other systems are in South America: São Paulo and Recife. In both metropolitan regions, buses carry the most passengers. For regions with area over 5,000 km^2, a rail system carries the most passengers in 75% of them. Only in São Paulo is this not the case.

Of the regions with population greater than 5 million people, 88% have a metro with extension greater than 100 km. Again São Paulo is the exception. This particularly contrasts with Mexico City, also located in a developing country in Latin America.

Comparison of the demographic density and extent of the rail system shows that all the regions with density over 5,000 people/km^2 have a rail system extending more than 100 km and those with population density over 9,000 people/km^2 have rail systems extending more than 500 km.

Analysis of the population density and the RFI shows that 67% of the regions with density greater than 5,000 people/km^2 have an RFI higher than 20. All those with density over 10,000 people/km^2 have an RFI greater than 60.

Analysis of the fare systems of the regions shows that in developing countries there is no predominance among the fare systems: two (São Paulo and Santiago) have a flat fare system, one has a zone-based system (Recife) and another a service-based system (Mexico City). Among developed countries, 67% of the metropolitan regions in Europe have a zone-based system, the exceptions being Brussels and Moscow, which have a flat fare system. All the systems in Asia charge fares based on distance. It is interesting to observe that Brussels, with a flat fare system and an RFI of 17, and Porto, with a zone-based fare system and an RFI equal to 30, have opposite characteristics: Brussels has a smaller area and greater population density than Porto. Among the regions that use a zone-based system, Paris has the highest RFI (33), while for those with a flat fare system New York-New Jersey has the highest RFI (53). Among the Asian regions, Seoul has the highest RFI (64). In North America, New York-New Jersey uses a flat fare system, while in Australia, Melbourne uses a zone-based system. The RFI of New York-New Jersey 2.8 higher than that of Melbourne.

A comparison of the RFI values shows that São Paulo has the lowest value (10) and Seoul the highest (64). This fact means that the average monthly salary users in São Paulo can cover 10 times the cost of commuting during the month, while the average monthly earnings of users in Seoul would cover 64 times the cost of commuting. Consequently, a typical resident of Seoul has the ability to use public transportation much more than a typical resident of São Paulo.

Comparison of fare integration and the fare system of the chosen places, Mexico City is the only one that has fare integration utilizing a service-based fare system. All the metropolitan regions in Europe (where zone-based fare systems predominate) have total fare integration. Outside Europe, those that have partial fare integration have a fare system based on distance (Tokyo) and flat fare (São Paulo and New York-New Jersey).

We should point out that in Latin America in general the systems do not encompass all means of transport. Instead, the network of each transport mode (bus, light rail, metro, and commuter train) are independent, with no single entity responsible for planning, operation and determining the minimum criteria for supply of service to the population.

PROPOSED METHOD

This section presents the procedure proposed to establish the best fare system to be implemented in a given locale, by applying a mathematical model.

According to the data in Table 2 and the analyses carried out previously, plus an analysis of the graphs plotted for each criterion[2], a relation can be perceived between the $RFI_{observed}$, the extension of the rail system and population density of the region. In this analysis we did not use the total extension of the transit system (all modes) because it was not possible to obtain the extension of the bus networks in all the metropolitan regions selected.

We used multiple linear regression to determine a mathematical model to provide the $RFI_{estimated}$ for each urban region under analysis.

Multiple linear regression:
$y_i = a + b_1X_1 + b_2X_2 + ... + b_nXn_i$
Where:
$y_i = RFI_{observed}$
X_k = variables chosen as determinants

Mathematical model adopted:

$$y = a + b_1 \text{ (demographic density)} + b_2 \text{ (rail line extension)},$$

where y is the $RFI_{estimated}$ of the region analyzed, and a, b_1 and b_2 are coefficients, which are determined after calculating the regression using Microsoft Excel.

The Appendix contains a summary of the results obtained from the linear regression.

[2] We prepared pair-wise graphs between the following characteristics: area, demographic density, rail line extension, number of passengers carried by rail, average monthly income, basic monthly fare and $RFI_{observed}$.

Analysis of the summary of the statistical results provided by Microsoft Excel leads to the following equation:

$$y = 7.3013 + 0.0025\ X_1 + 0.0105\ X_2.$$

This equation is valid because it has a high value of R^2 (above 50%), corresponding to 63% of the association between the dependent variable (y) and the two dependent variables, and suitable t-statistics for the variables X_1 and X_2. The t-value obtained in relation to the intercept is less than 2. Therefore, we performed tests at lower confidence intervals (90%, 85%,...), but the t-value didn't change(1.457, 1.457,...), so we did not consider them.

We also carried out a multiple linear regression considering an intercept of zero, but the results obtained presented negative side effects, with a high F-value of 47 (greater than $F_{critical}$ = 22).

Equation defined:

$$y = 7.3013 + 0.0025*\text{demographic dens} + 0.0105*\text{rail line extension} \quad \text{Equation 1}$$

t = 1.233 t = 3.695 t = 2.657

Data that contributed to determination of the procedure

Tables 3, 4 and 5 present the data on demographic density, extension of the rail system, observed RFI, fare integration, monthly pass and predominant fare system. The characteristics of the variables are:

1. Demographic density: The population density of each urban region, according to the data presented previously.
2. Network extension: The sum all the rail systems (metro, train and/or light rail), according to the data presented previously.
3. RFI (relative fare index): Calculated as the average monthly income of the population divided by the weighted average monthly fare.
4. Fare integration:
   - ✓ Total: There is a single ticket / pass allowing riders to use all the transport systems existing in a region.
   - ✓ Partial: There is a single ticket / pass allowing riders to use certain transport systems in a region.
   - ✓ None: There is no single ticket / pass allowing riders to use all the transports systems in a region.

5. Monthly pass: This variable determines whether the region analyzed has a monthly pass that covers all the transports systems.

6. Predominant fare system: This variable determines what each region's predominant fare system is.

Location	Demog. density (inhab/km^2)	Ext. (km) rail	RFI observed	Fare integration	Month pass ?	Predominant Fare System
São Paulo Metropolitan area	2,450	318.8	10	Partial	No	Flat Fare[1]
Recife metropolitan area	1,320	60.8	11	Total	No	Zone
Santiago	5,325	149.4	15	Total	No	Flat Fare
Mexico City (DF)	5,965	202.7	16	None	No	Service
Brussels Capital region	4,554	339.6	17	Total	Yes	Zone
Melbourne metropolitan area	443	631.0	19	Total	Yes	Zone
Moscow	9,660	1476.0	22	Total	Yes	Flat Fare[2]
Greater London	4,757	1253.0	23	Total	Yes	Zone
Madrid	5,304	321.0	27	Total	Yes	Zone
Porto metropolitan area	1,571	112.3	30	Total	Yes	Zone
Ile-de-France	956	1645.8	33	Total	Yes	Zone
New York - New Jersey metropolitan area	1,700	3124.7	53	Partial	No	Flat Fare[3]
Tokyo city	13,786	600.0	62	Partial	No	Distance
Seoul metropolitan area	17,019	532.9	64	Total	Yes	Distance

1. We considered São Paulo to have a flat fare system, since 55.07% of the trips are by flat fare.
2. We also considered Moscow to have a flat fare system, since 57% of the trips are taken by this system.
3. For the NY-NJ Metropolitan Region we considered the fare system to be that of New York City (flat fare), since 89.34% of the trips in the region are by single fare

As shown in Table 4, the urban regions in developing countries do not have a predominant fare system.

Among the urban regions in developed countries (Table 5), most of those in Europe have a zone-based fare system and total fare integration, monthly passes and intermediate RFI values. Those in Asia have the highest RFI values, high population density and virtually all have integrated systems.

Table 3: Fare systems and RFI of metropolitan regions in developing countries

Developing countries

Location	Demog. density ($inhab/km^2$)	Ext. (km) rail	RFI observed	Fare integration	Month pass ?	Predominant Fare System
São Paulo Metropolitan area	2,450	318.8	10	Partial	No	Flat Fare[1]
Recife metropolitan area	1,320	60.8	11	Total	No	Zone
Santiago	5,325	149.4	15	Total	No	Flat Fare
Mexico City (DF)	5,965	202.7	16	None	No	Service

Table 4: Fare systems and RFI of metropolitan regions in developed countries.

Países desenvolvidos

Location	Demog. density ($inhab/km^2$)	Ext. (km) rail	RFI observed	Fare integration	Month pass ?	Predominant Fare System
Brussels Capital region	4,554	339.6	17	Total	Yes	Flat Fare
Melbourne metropolitan area	443	631.0	19	Total	Yes	Zone
Moscow	9,660	1476.0	22	Total	Yes	Flat Fare[2]
Greater London	4,757	1253.0	23	Total	Yes	Zone
Madrid	5,304	321.0	27	Total	Yes	Zone
Porto metropolitan area	1,571	112.3	30	Total	Yes	Zone
Ile-de-France	956	1645.8	33	Total	Yes	Zone
New York - New Jersey metropolitan area	1,700	3124.7	53	Partial	No	Flat Fare[3]
Tokyo city	13,786	600.0	62	Partial	No	Distance
Seoul metropolitan area	17,019	532.9	64	Total	Yes	Distance

1. We considered São Paulo to have a flat fare system, since 55.07% of the trips are by flat fare.
2. We also considered Moscow to have a flat fare system, since 57% of the trips are taken by this system.
3. For the NY-NJ Metropolitan Region we considered the fare system to be that of New York City (flat fare), since 89.34% of the trips in the region are by single fare

RESULTS

The routine described below determines the best fare system to adopt:

1. Calculate the demographic density and obtain the rail system extension.
2. Apply the rail system extension and demographic density in Equation 1.

If the region belongs to a developed country:

1. and has an RFI between 17 and 33, then the best fare system is zone based;
2. and has an RFI between 34 and 59, then the best fare system is flat fare;
3. and has an RFI greater than 60, then the best fare system is distance based.

If the region belongs to a developing country:

1. and has an $RFI_{estimated}$ less than 17 and a demographic density between 3,500 and 5,500 people/km^2, the best fare system is flat fare;
2. and has an $RFI_{estimated}$ less than 17 and a demographic density lower than 3,499 people/km^2, the best fare system is zone based;
3. and has an RFI greater to 17, the government should decide which system to adopt, based on the data exemplified in Tables 38 and 40.

CONCLUSIONS

The main objective of this study was to develop a method for initial evaluation in the process of deciding on the best fare system to implement in a determined urban region.

From the literature review, research of socioeconomic data and data on mass transit systems, we determined a database made up of some representative metropolitan regions: Federal District of Mexico, New York-New Jersey Metropolitan Region, Recife Metropolitan Region, City of Santiago, São Paulo Metropolitan Region, Brussels Capital Region, Greater London, City of Madrid, City of Moscow, Ile-de-France or Greater Paris, Porto Metropolitan Area, Melbourne Metropolitan Area, Seoul Metropolitan Area and the City of Tokyo. We found that:

1. In the regions that have a zone-based fare system, the extension of the rail network is greater than 300 km (except for Porto, which due to lack of data on the rail network does not meet this value) and the RFI ranges from 13 to 33. Those with populations over 7.5 million also transport more than 1.5 billion passengers a year.

2. In the regions where the fare system is mainly based on distance, the RFI is greater than 60, the rail network is extensive and the population density is greater than 10,000 people/km^2.

3. The smallest RFI values and the rail network extensions are in regions in developing countries.

4. Most of the regions analyzed have a rail network (metro, light rail and/or commuter train) that carries the most passengers, while São Paulo stands out by having a bus system that does this. This situation is typical of Brazilian cities, which are notable for their relative lack of rail infrastructure and low quality of service.

5. With respect to the metro (generally subway) network, practically all the regions analyzed that have a population over 10 million people also have a network over 200 km. However, São Paulo is again an exception. It has well over 10 million people but its metro system only extends 61.3 km. This is only 16% of the metro network's extension in the New York-New Jersey Metropolitan Region.

6. São Paulo also has the lowest RFI value (10), while Seoul has the highest (64). This means that a typical rider's monthly income will only cover 10 times the cost of commuting per month, while in Seoul this figure is 64. The regions in developing countries have the lowest RFI values, and of developed countries, those with the highest RFI values are located mainly in Asia. The low RFI values in developing countries reflect the high relative fare cost. This is a problem that needs addressing by policymakers, by lowering fares, increasing income and/or taking other measures to enhance public mobility.

7. The fare amount in relation to average income in São Paulo is disproportional compared to the other regions analyzed.

The analysis of the data and graphs plotted for each criterion showed there is a connection between the RFI, extension of the rail network and population density of the regions studied. We used multiple linear regression, with demographic density and rail network extension as the control variables, to obtain an equation equivalent to the estimated RFI.

Depending on the RFI value obtained by this mathematical model and the location of the region analyzed (developed versus developing country), we determined the possible changes in the current system that can be implemented to attain an optimal fare scheme.

In light of the objective of this study (to develop a method to establish the best fare system to be implemented in a given metropolitan region), we believe the method developed fills this purpose.

The planning and preparation of a fare system coherent with the characteristics of the place it will be implemented is a key element to the successful development of the area and greater mobility of the population.

Brazilian cities have particularly poor rail networks (both in extension and service levels), and instead typically rely on chaotic bus systems for most mass transit needs. These factors, combined with the high relative fares, hinder commuting. Riders typically must use two or more means of transport, generally paying separate fares.

One drawback in this respect in Brazil is the fact that the Federal Constitution (Art. 30, V) defines the organization of mass transit as a municipal responsibility, but most large metropolitan regions stretch over more than one municipality. This makes coordination to develop rationally integrated mass transit systems that facilitate public mobility difficult. The experience of other countries in this respect, particularly in Europe, Asia and North America, shows that this problem can be successfully addressed with the proper political determination.

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